Wednesday, July 25, 2012
8:45 AM

Amplitude Modulation

Key Equation for DSB-SC

$$\mathrm{LPF}\left\{\underbrace{\Big(\underset{\text{message}}{m(t)}\ \underbrace{\times\sqrt{2}\ \overbrace{\cos(2\pi f_c t)}^{\text{carrier}}}_{\text{modulation}}\Big)}_{x(t)\ \leftarrow\ \text{transmitted signal}}\times\sqrt{2}\cos(2\pi f_c t)\right\} = m(t)$$

Things that could go wrong ...

1. Frequency / phase offset
2. Time delay

Solution:
- synchronization (via PLL)
- put some extra info into the transmitted signal to help the receiver.

Fourier Series

Periodic signal $r(t)$ with period T_0 can be expanded into

$$r(t) = \underbrace{\sum_k c_k e^{j2\pi k f_0 t}}_{\text{writing it in this form suggest that its Fourier transform is}}$$

$$R(f) = \sum_k c_k \delta(f - kf_0)$$ ← sum of scaled and shifted impulses in the freq. domain.

The coefficient c_k can be found by $c_k = \frac{1}{T_0} R_{T_0}(kf_0)$.

In particular, first we consider a resticted version $r_{T_0}(t)$ of $r(t)$.

The restriction simply means we will use only one period of $r(t)$.

This can be

$$r_{T_0}(t) = \begin{cases} r(t), & |t| \le T_0/2 \\ 0, & \text{otherwise.} \end{cases}$$

Next, we find the Fourier transform of $r_{T_0}(t)$ which we denote by $R_{T_0}(f)$

Then, we scaled the result by $\frac{1}{T_0}$ to get $\frac{1}{T_0}R_{T_0}(f)$.

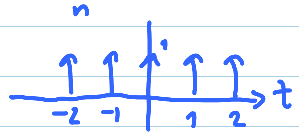
Finally, we evaluate the expression at $f=kf_0$.

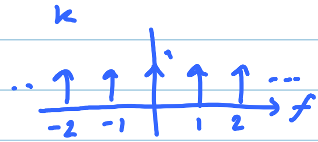
Example:

$$\sum_n \delta(t-nT_0) = \frac{1}{T_0}\sum_k e^{j2\pi k f_0 t} \xrightarrow{\mathcal{F}} \frac{1}{T_0}\sum_k \delta(f-kf_0)$$

Special case: when $T_0=1$, we have

$$\sum_n \delta(t-n) \xrightarrow{\mathcal{F}} \sum_k \delta(f-k)$$





Fourier Series and Fourier Transform: the comparison

Recall that $|X(f)|^2$ is the energy density

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df \xrightarrow{\mathcal{F}} X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

complex exponential at freq. f

complex exponential at freq. $f=kf_0$

$$r(t) = \sum_k c_k e^{j2\pi k f_0 t} \xrightarrow{\mathcal{F}} R(f) = \sum_k c_k\, \delta(f-kf_0)$$

coefficient

tell you the amount of the signal energy ($|c_k|^2$) at freq. $f=kf_0$